Programming Languages: Imperative Program Construction
Practicals 4: Hoare Logic and Weakest Precondition: Loop

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1. Prove the correctness of the following program:

   con $N : Int\ \{N \geqslant 0\}$
   var $x, y : Int$
   $x, y := 0, 1$
   do $x \neq N \rightarrow x, y := x + 1, y + y$ **od**
   $\{y = 2^N\}$

2. Prove the correctness of the following program:

   con $A, B : Int\ \{A \geqslant 0\}$
   var $r, n : Int$
   $r, a := 0, 0$
   do $a \neq A \rightarrow r, a := r + B, a + 1$ **od**
   $\{r = A \times B\}$

3. Prove the correctness of the following program:

   con $N : Int\ \{N \geqslant 0\}$
   con $A :$ **array** $[0..N)$ **of** Int
   var $n, x : Int$
   $x, n := 0, 0$
   do $n \neq N \rightarrow x, n := x + A[n], n + 1$ **od**
   $\{x = \langle \Sigma i : 0 \leqslant i < N : A[i] \rangle\}$

4. Prove the correctness of the following program:

   con $N : Int\ \{N \geqslant 0\}$
   var $y : Int$
   $y := 1$
   do $y < N \rightarrow y := y + y$ **od**
   $\{y \geqslant N \wedge \langle \exists k : k \geqslant 0 : y = 2^k \rangle\}$

5. Given integers $N \geqslant 0$ and $M > 0$, the following program computes integral division N / M. Prove its correctness.

$$
\begin{array}{l}
\textbf{con } N, M : \mathit{Int}\ \{N \geqslant 0 \wedge M > 0\} \\
\textbf{var } l, r : \mathit{Int} \\
l, r := 0, N + 1 \\
\textbf{do } l + 1 \neq r \rightarrow \\
\quad \textbf{if } ((l + r) \mathbin{/} 2) \times M \leqslant N \rightarrow l := (l + r) \mathbin{/} 2 \\
\quad \mid ((l + r) \mathbin{/} 2) \times M > N \rightarrow r := (l + r) \mathbin{/} 2 \\
\quad \textbf{fi} \\
\textbf{od} \\
\{l \times M \leqslant N < (l + 1) \times M\}
\end{array}
$$

6. The following program non-deterministically computes x and y such that $x \times y = N$. Prove:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 1\} \\
\textbf{var } p, x, y : \mathit{Int} \\
p, x, y := N - 1, 1, 1 \\
\{N = x \times y + p \wedge \ldots\} \\
\textbf{do } p \neq 0 \rightarrow \\
\quad \textbf{if } \quad p \textbf{ mod } x = 0 \rightarrow y, p := y + 1, p - x \\
\qquad \mid p \textbf{ mod } y = 0 \rightarrow x, p := x + 1, p - y \\
\quad \textbf{fi} \\
\textbf{od} \\
\{x \times y = N\}
\end{array}
$$

7. Prove the correctness of the following program:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 0\} \\
\textbf{var } x, y : \mathit{Int} \\
x, y := 0, 0 \\
\textbf{do } x \neq 0 \ \rightarrow x := x - 1 \\
\mid \quad y \neq N \rightarrow x, y := x + 1, y + 1 \\
\textbf{od} \\
\{x = 0 \wedge y = N\}
\end{array}
$$

8. Prove the correctness of the following program:

$$
\begin{array}{l}
\textbf{con } N : \mathit{Int}\ \{N \geqslant 0\} \\
\textbf{var } x, y : \mathit{Int} \\
x, y := 0, 0 \\
\textbf{do } x \neq 0 \ \rightarrow x := x - 1 \\
\mid \quad y \neq N \rightarrow x, y := N, y + 1 \\
\textbf{od} \\
\{x = 0 \wedge y = N\}
\end{array}
$$