

Programming Languages: Imperative Program Construction

Practicals 5: Loop Constuction I

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1. Derive a program for the computation of square root.

```
con  $N : Int \{0 \leq N\}$ 
var  $x : Int$ 
squareroot
 $\{x^2 \leq N < (x + 1)^2\}$  .
```

2. For each implication below, find a substitution (on variables) such that the implication holds. Note:

- Names starting with small letters (x, a, b , etc) are variables, while A, B , and C are constants. E denotes an expression.
- We assume that all variables and constants are Int .
- For some questions, there could be more than one substitutions that work.

- (a) $(x = 2 \times E)[? \backslash ?] \Leftarrow x = E$, where x does not occur free in E .
- (b) $(x = 2 \times E + A)[? \backslash ?] \Leftarrow x = E$, where x does not occur free in E .
- (c) $(x = f E)[? \backslash ?] \Leftarrow x = E$, for some function f . Again, x does not occur free in E .
- (d) $(x = A)[? \backslash ?] \Leftarrow x = 2 \times A + B$.
- (e) $(A = 2 \times b \times x + c)[? \backslash ?] \Leftarrow A = b \times x + c \wedge \dots$ You may need to discover an additional condition in ... to make the implication valid.
- (f) $(A = B \times x + B + C)[? \backslash ?] \Leftarrow A = B \times x + C$.
- (g) $(A = B \times x / 2 + 2 \times C)[? \backslash ?] \Leftarrow A = B \times x + C \wedge \dots$ You will need a side condition. Note that (\times) and $(/)$ are left-associative. That is, $B \times X / C$ is interpreted as $(B \times X) / C$.

3. **The Zune problem.** Let D be the number of days since 1st January 1980. What is the current year? Assume that there exists a function $daysInYear : Int \rightarrow Int$ such that $daysInYear i$, with $i \geq 1980$, yields the number of days in year i , which is always a positive number. Derive a program having two variables y and d such that, upon termination, y is the current year, and d is the number of days since the beginning of this year.

- (a) How would you specify the problem? The specification may look like:

```
con  $D : Int \{0 \leq D\}$ 
var  $y, d : Int$ 
zune
 $\{???\}$ 
```

What would you put as the postcondition? In this postcondition, is 1st January 1980 day 0 or 1?

- (b) Derive the program.

4. Assuming that $-\infty$ is the identity element of (\uparrow) . Derive a solution for:

```
con  $N : \text{Int} \{N \geq 0\}$   
con  $A : \text{array} [0..N) \text{ of } \text{Int}$   
var  $r : \text{Int}$   
 $S$   
 $\{r = \langle \uparrow i : 0 \leq i < N : A[i] \rangle\} .$ 
```

5. Derive a solution for:

```
con  $N, X : \text{Int} \{0 \leq N\}$   
con  $A : \text{array} [0..N) \text{ of } \text{Int}$   
var  $r : \text{Int}$   
 $S$   
 $\{r = \langle \sum i : 0 \leq i < N : A[i] \times X^i \rangle\} .$ 
```